Phonon Escape Time Deduced From the Time of Nucleation of Hot Spots in Superconducting Niobium Filaments

K. Harrabi, F. Oktasendra, K. Gasmi, G. R. Berdiyorov, A. Mekki, and J. P. Maneval

***Abstract*—We have observed the voltage response of superconducting niobium strips to overcritical ($I > I_c$) step pulses of electrical current. The resistive response appears after a certain delay time t_d function of the temperature and of the ratio $I/I_c(T)$, which can be analyzed through a time-dependent Ginzburg–Landau equation, along the method introduced by Tinkham. The experimental data can be fitted by using an effective gap relaxation time of a few nanoseconds, independently of the sample widths and of the temperature. Assuming proportionality to sample thickness, this indicates a phonon escape time of about 8 ps per nanometer of thickness for a Nb film dc sputtered on polished crystalline Al_2O_3.**

***Index Terms*—Critical current density, non-equilibrium superconductivity, superconducting thin film.**

PACS numbers
74.40.Gh: Nonequilibrium superconductivity
74.25.Sv: Critical currents
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74.25.Kc: Phonons.

I. Introduction

ONE of the intriguing phenomena of non-equilibrium superconductivity remains the formation of resistive spots preserving the superconducting order despite the periodic destruction of the order parameter in their center. The dissipation originates from the creation of the excess quasi-particles [1]. The phase-slip center (PSC) in one-dimensional filaments [2] is the locus of alternance in time of the superconducting and normal components of the electrical current. According to the Skocpol, Beasley, Tinkham model [3], [4], the phase difference of the order parameter slips by a quantum of 2π each time the superconducting gap drops to zero. This cycle generates non-equilibrium quasi-particles which diffuse over an inelastic diffusion length setting the dimension of the PSC. However, numerous generated quasiparticles can overheat the filament leading to the formation of a localized normal state or hot spot (HS) [2]. In one-dimensional Al nanowires [5], for example, a single PSC can switch the wire from the superconducting to normal state. The observation of HS formation preceded by the nucleation of PSC has also been reported in wider superconducting structures [6]–[8].

A HS is essentially a normal zone maintained above the critical temperature by the Joule effect [4]. The heat generated in the normal zone is transferred along the filament by thermal conduction as well as evacuated to the substrate by surface heat transfer. The latter occurs by means of phonon radiation through the interface [9]. In this paper, we study the characteristic time needed by the phonons to escape from the film to the substrate. In single photon detectors, the phonon escape time is an important parameter since it contributes to determining the recovery time of the detection process, for instance, in relation with its application to optical communication protocols [10].

II. Samples and Experimental Setup

The Nb thin films, 80 nm in thickness and 3 mm in length, used in our experiment were grown on R-cut sapphire substrates and prepared by STAR-Cryoelectronics (NM, USA). The pattern consists of a linear bridge and of two lateral voltage probes which were obtained via standard photo-lithographic processes and ion milling. Four 100-nm-thick gold pads were evaporated to be used as electrodes. Then, a series of measurements were performed on three samples labeled FNbW3, FNbW5 and FNbW10 having different widths of 3, 5, and 10 μm, respectively. From R-T measurement, we obtained the transition temperatures, $T_c = 8.2, 8.0,$ and 8.2 K and resistivities at temperature of 10 K, $\rho_{10\mathrm{K}} = 9.57, 8.09,$ and 7.35 $\mu\Omega$.cm, respectively. On a sample taken from the same wafer, the residual resistance ratio $\rho_{300\mathrm{K}}/\rho_{10\mathrm{K}}$ was found to be RRR $=$ 3.13. According to a standard method recalled in [8], one can deduce the electron elastic mean free path $l_e \sim 5$ nm at 10 K, much shorter than the coherence length of the pure material, which places our samples in the dirty limit.

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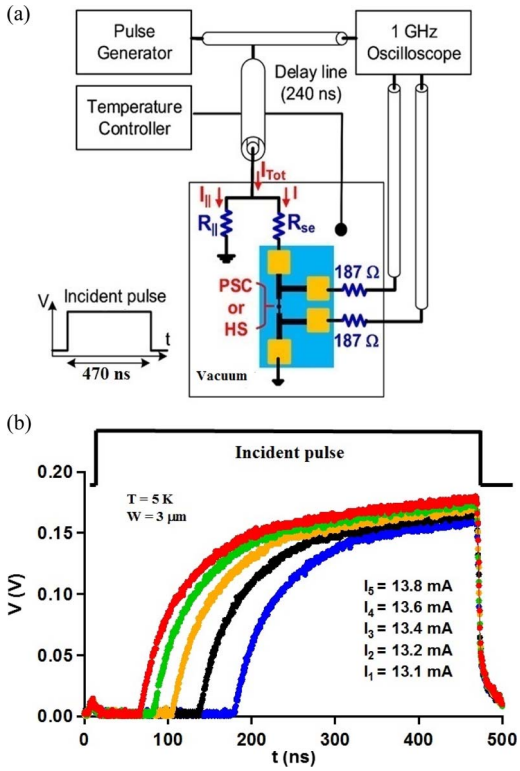

Fig. 1. (a) Experimental setup used for pulse measurements. A 240-ns delay line separates (in the time domain) the incident pulse from the reflected one. Several metallic resistances ($R_{\rm se} = 187\ \Omega$; $R_{\|} = 67\ \Omega$) aim at keeping the current almost constant during the pulse. (b) Oscilloscope traces of the voltage, with respect to ground, collected at the upper side electrode. The origin of the times, defined by the onset of the current pulse (schematic top trace), coincides with the small inductive peak common to all traces. The delay time t_d is a rapidly decreasing function of the current.

Our experiments were performed in a cryo-free closed-cycle cryostat in the range 4 K to T_c. To measure the voltage response of the sample, a rectangular current pulse having 470 ns duration was sent from the signal generator to the sample through a 50 Ω coaxial cable (Fig. 1(a)). In order to achieve constant-current biasing throughout the duration of the pulse, a large series resistance $R_{\rm se}$ was inserted. The $R_{\|}$ resistor completes the circuit so as to present a resultant impedance of 50 Ω to the incident signal in the superconducting state ($R = 0$ or S state). By assuming a lossless coaxial cables and negligible contact resistances, we obtained in the S state $I = I_{\rm tot}R_{\|}/(R_{\|} + R_{\rm se})$, where $I_{\rm tot} = (V_i/Z)$ ($Z = 50\ \Omega$), and V_i is the incident voltage. The addition of 187 Ω connected in series with the lateral probes was meant to avoid the current leak to the output circuitry. The closed-cycle cryostat was equipped with a cryogenic temperature controller system programmed for closed loop temperature control in PID mode.

III. Nucleation of a Hot Spot by a Current Pulse

When the superconducting filament is excited by an overcritical current, there are two possible responses in the absence of flux flow, which is the case here. The PSC is a localized S structure ($T < T_c$) which manifests itself by a constant voltage. The hotspot (HS) is a normal zone ($T > T_c$) [12] steadily growing

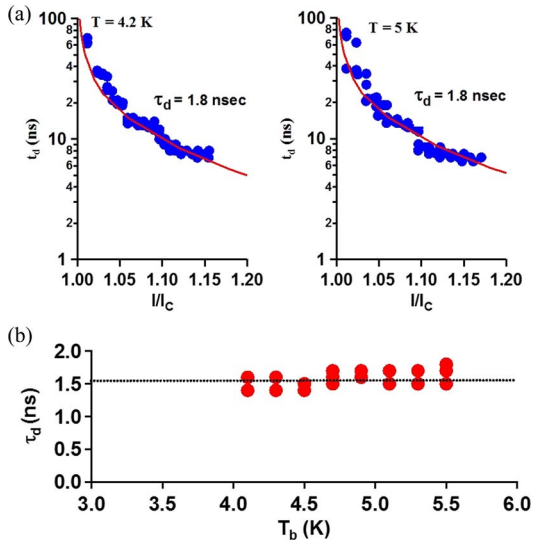

Fig. 2. Delay time t_d (log scale) as a function of the reduced current I/I_c for several fixed temperatures in sample FNbW3. The fitting procedure consists in sliding vertically the theoretical function to fit the experimental data.

in constant-current biasing, whose voltage drop however tends to saturate due to current weakening. The signals of Fig. 1(b) have the characteristics of a HS, but the important point is that the delay time t_d before the appearance of a resistive voltage has the same underlying cause [8], the collapse of the order parameter function. So, we will analyze the delay times of HSs by using the same theoretical approach [13], [15] as for the S state.

IV. Temperature Dependence of the Delay Time

The time interval of zero voltage which precedes the appearance of a resistive state in a superconducting strip was discovered by Pals and Wolter [13] and interpreted on the basis of the Time-Dependent Ginzburg-Landau (TDGL) theory. In a one-dimensional geometry, the time it takes to suppress the superconducting order parameter f takes the form

$$t_d\left(\frac{I}{I_c}\right) = \tau_d \int_0^1 \frac{2f^4 df}{\frac{4}{27}\left(\frac{I}{I_c}\right)^2 - f^4 + f^6} \qquad (1)$$

where the prefactor τ_d is related to the inelastic electron-phonon relaxation time τ_E in the normal state close to the critical temperature. Soon after, Tinkham proposed a modified theory [15] where τ_d is the temperature dependent gap relaxation time, and the integrand of Eq. (1) is also modified. As a result, the delay time became dependent upon the two variables T/T_c and I/I_c. The predictions were confirmed by observations on indium strips [16]. In the case of YBCO epitaxial films [7], [11], the experiments proved compatible features (a) the prefactor τ_d is constant up to about the $T_c/2$ and slowly increasing up to T_c and (b) at elevated temperatures (near T_c), it tends to reproduce the bolometric response time determined in conventional ways. From these observations, it was concluded [11] that in a number of cases, the prefactor τ_d could be replaced by the cooling time $\tau_{\rm cool}$, at least in cases where the evacuation of the heat is the

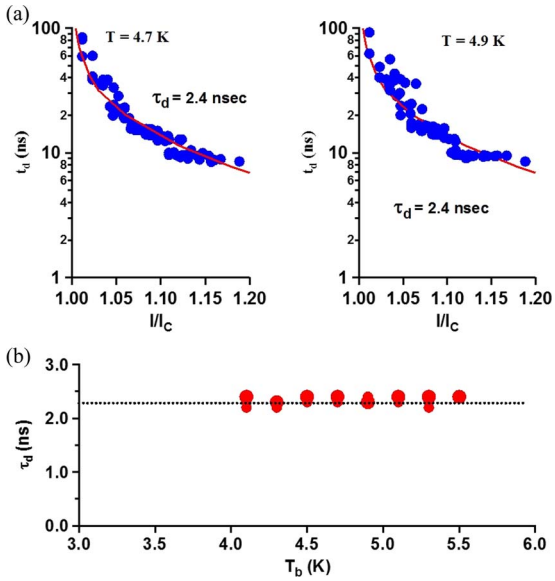

Fig. 3. (a) Delay time versus I/I_c fitted with Tinkham's TDGL theory for different temperatures (continuous traces) of sample FNbW5. (b) Temperature dependence of the heat escape time.

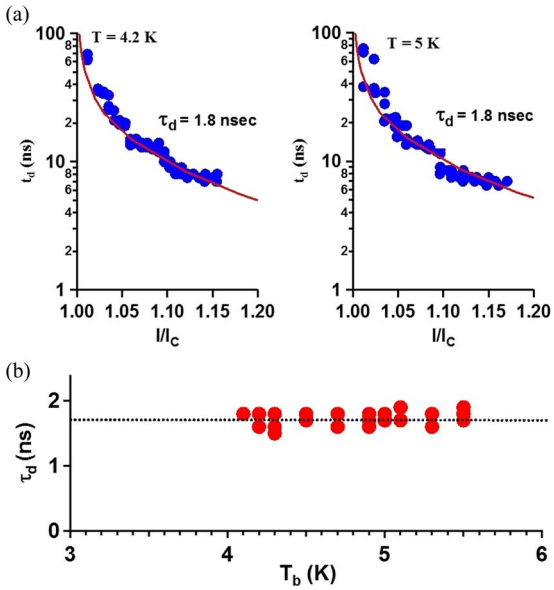

Fig. 4. (a) Delay time versus I/I_c fitted with Tinkham's TDGL theory for different temperatures (continuous traces) of sample FNbW10. (b) Temperature dependence of the heat escape time.

limiting factor for electron energy relaxation. Consistent interpretations of the experiments on YBCO [7] and NbTiN [18] were proposed. We apply the same principles in the following treatment of niobium data.

Fig. 2(a) shows the data of t_d as a function of I/I_c at the two temperatures 4.2 K and 5.0 K on sample FNbW3. The two are similar and can be fitted by Tinkham functions with the same prefactor $\tau_d = 1.6$ ns. Over a wider temperature range, Fig. 2(b) shows a slight tendency of τ_d to increase with temperature.

The plots of Figs. 3(a), (b) and 4(a), (b) applying to samples FNbW5 and FNbW10 confirm the quasi-invariability of τ_d upon the temperature, even if there is some scatter from sample

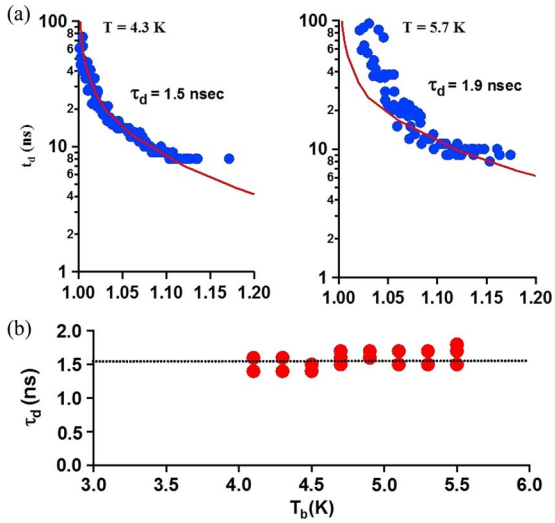

Fig. 5. (a) Delay time versus I/I_c and fitting (red continuous trace) with Tinkham's TDGL equation at $T = 4.2$ K (left) and 5.7 K (right) of sample FNbW10. (b) Temperature dependence of the heat escape time (cooling time).

to sample. If we adopt the conclusions of the work done on YBCO thin films, the prefactor τ_d might represent the cooling time of the film on its substrate. By taking the mean value of τ_d over Figs. 2–4 (ignoring the somewhat erratic data of Fig. 5), we find $\tau_d = 2.0$ ns for a common thickness 80 nm.

Our experimental findings of PSC nucleation times, performed on $YBa_2Cu_3O_7$ [11] and Nb [8] films far from T_c (typically $T_c/2$), concluded to the quasi-independence of the prefactor τ_d upon the temperature. Therefore, it was proposed that τ_d be identified with the film thermal relaxation (or cooling) time $\tau_{\rm cool}$. That characteristic time is related to the phonon escape time by the equation $\tau_{\rm esc} = C_\phi \tau_{\rm cool}/(C_\phi + C_{\rm eS})$, where C_ϕ and $C_{\rm eS}$ are, respectively the phonon specific heat and the electron specific heat in the superconducting (S) state. By injecting the appropriate specifications for niobium [8], that is $C_\phi(T) = 12.45\ {\rm T}^3\ {\rm Jm}^{-3}{\rm K}^{-4}$ and $C_{\rm eS}(T) = 26.5\ {\rm T}^3\ {\rm Jm}^{-3}{\rm K}^{-4}$, and using $\tau_{\rm cool} = \tau_d = 2.0$ ns, one obtains $\tau_{\rm esc} = 0.64$ ns and $\tau_{\rm esc}/b = 8$ ps/nm for a Nb film of thickness $b = 80$ nm. That is smaller than the result $\tau_{\rm esc} \sim 0.3$ ns for $b = 20$ nm obtained by N. Bluzer [21] in a pulse photo-resistance experiment, performed at higher temperatures in the normal state however. On the other hand, Semenov *et al.* [22] obtained the same ratio $\tau_{\rm esc}/b = 8$ ps/nm for a 20 nm thick NbN film.

A simple approach to the phonon escape time is via the acoustic mismatch radiation model [23], in which film and substrate are treated as two communicating phonon blackbodies. For a single mode, or polarization, the mean escape time is $\tau_{\rm esc} = 4b/\eta c$, where c is the velocity of sound in the film and η is the outward transmission coefficient. The factor of $4 = 2 \times 2$ accounts for the two opposite directions of propagation in the film, plus an angular integration over the squared-cosines. In view of the three phonon polarizations, the escape time becomes

$$\tau_{\rm esc} = 4b\langle c^{-3}\rangle/\langle \eta c^{-2}\rangle \qquad (2)$$

where a bracket $\langle \ldots \rangle$ indicates a summation over the three modes. With the numbers provided by S. Kaplan [23] for the

couple Nb-on-sapphire, that is $\eta = 0.38$ and 0.091, $c = 5.14$, and 2.17 km/s, for the longitudinal and transverse phonons, respectively, one obtains the ratio $\tau_{\rm esc}/b = 15$ ps/nm, about twice as much as our experimental findings.

Because of the smallness of the temperature range investigated, it is not possible to assign to a given physical mechanism the slight, but systematic, increase of τ_d, on all samples.

V. Conclusion

By interpreting through a TDGL analysis the time of appearance of the first resistive response to an over-critical current pulse, we have obtained the thermal relaxation time of an 80-nm niobium film dc-sputtered on sapphire. The narrow domain investigated (4 to 5.5 K) does not allow noting anything else but a slight tendency to increase with the temperature. The associated phonon escape time, which follows $\tau_{\rm esc}/b = 8$ ps/nm ($b =$ thickness) is in better than order-of-magnitude agreement with both the acoustic mismatch model and other independent experimental determinations.

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